

On Accounting “Capital” in Horizontal Innovation Models

Abstract

I identify and correct some logical problems of the 1990 Romer model of horizontal innovation. The model’s setting where all the produced inputs are durable is inconsistent with the proposed Dixit-Stiglitz production function; also, Romer’s “accounting measure of capital” (and its variants in the literature) is not a theoretically proper measure on which the rate of interest is to be calculated. The correction is straightforward. However, a more fundamental issue is at stake: the structure of horizontal innovation models necessitates consideration of the problem of value, but the current literature deliberately assumes it away, generating internal inconsistency of various forms. (JEL O41)

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“[Z]ero tolerance for intellectual sloppiness”

(Paul Romer, in Snowdon and Vane 2005).

I. Introduction

Romer (1990) is a seminal paper which ushered in active research in endogenous growth theory in general and horizontal innovation models in particular. One of the building blocks of the model presented in that paper is the concept of the “accounting measure of capital.” The importance of this concept is attested to by the fact that it appears in *all* horizontal innovation models, albeit generated through different procedures and taking different forms such as the “assets” of the economy in Barro and Sala-i-Martin (2004, Ch. 6) or the “financial wealth” in Jones (2005).¹ It plays a crucial analytical role: to represent a state variable in the dynamic utility maximization problem.

After a brief exposition in Section II of the concept of the “accounting measure of capital” proposed by Romer (1990), I will argue in Section III that the assumption that all inputs in the various sectors of production are durable is *inconsistent* with the production process envisaged in the Dixit-Stiglitz production function used in the same model. I will in Section IV suggest a way, among several alternatives, of correcting the model. It will turn out that the correction leaves the robustness of the model intact (the Appendix solves the model-consistently modified model). But this correction will, as is discussed in detail in Section V,

¹ See Gancia and Zilibotti (2005) for an excellent survey of horizontal innovation models, which distinguishes among the “baseline,” the “lab equipment” and the “labor for intermediates” models. The Romer model is representative of the “baseline” model.

lead to the recognition that though the “accounting measure of capital” (and its variants in the literature) is an ingenious device which makes the mathematical formulation of the model much simpler, it is, when looked at from the *theoretical* perspective, not a proper measure of capital in reference to which the rate of interest is to be calculated. Section VI concludes by way of reminding the reader that a fundamental issue is at stake in the use of the “accounting measure of capital” (and its variants). Multi-sector models such as Romer’s and, by extension, all horizontal innovation models are *inescapably* subject to the problem of *value*, but they (unwittingly or deliberately) ignore it. The embodiment of this ignorance is the “accounting measure of capital” (and its variants). The problem is that its use in the literature generates one or another form of internal inconsistency.² Horizontal innovation models have paid excessive price—the violation of some fundamental principles of economics—in return to the simplification of the models, for the violation is not necessary if one is willing to allow for a slight increase in the complexity of the models.

II. The “accounting measure of capital”

The Romer economy is specified as follows. The research sector (the R sector) produces designs, using the stock of previous designs (which are public goods) and human capital (H_A). The intermediate goods sector (the I sector) produces intermediate goods by means of designs and the final good: η units of the final good are required to produce one unit of an intermediate good. The final good sector (the Y sector) produces the final good by means of human capital (H_Y), labor and the stock of intermediate goods. The final good is either

² The case of Barro and Sala-i-Martin (2004, Ch. 6), an example of Gancia and Zilibotti’s (2005) “lab equipment” model, is worked out in a companion paper, which is available from the author on request.

consumed or foregone for investment in the I sector.

The stock of intermediate goods used in the production of the final good constitutes the “capital” of the Y sector. Romer defines the “accounting measure of capital” as follows:

In parallel with the usual one-sector model and in conformity with national income accounting conventions, it is useful to define an accounting measure of total capital K as cumulative foregone output. Thus $K(t)$ evolves according to the rule

$$\dot{K}(t) = Y(t) - C(t), \quad (1)$$

where $C(t)$ denotes aggregate consumption at time t [and $Y(t)$ the aggregate output of the final good] (Romer 1990, p. S82).

He continues:

Because it takes η units of foregone consumption to create one unit of any type of durable [intermediate good], this accounting measure K is related to the durable goods that are actually used in production by the rule $K = \eta \sum_{i=1}^{\infty} x_i = \eta \sum_{i=1}^A x_i$ (Romer 1990, p. S82),

where A is the number of the types of intermediate goods which are actually used and x_i is the physical quantity of the i th type of intermediate goods. It turns out that equilibrium is symmetric, so that $x_i = x, \forall i$; therefore, the “accounting measure of capital” is eventually calculated as

$$K(t) = \eta x A(t). \quad (2)$$

The key role of this measure of capital is to represent a state variable in the dynamic utility maximization problem. The representative household maximizes discounted utility over infinite lifetime on the basis of the following utility function:

$$U(C) = \frac{C^{1-\sigma} - 1}{1-\sigma} . \quad (3)$$

The resulting Euler equation is

$$\frac{\dot{C}}{C} = \frac{r - \rho}{\sigma} , \quad (4)$$

where r is the rate of interest, ρ the rate of time preference and σ^{-1} the elasticity of intertemporal substitution. Romer does not explicitly write down the constraints for the optimization problem which leads to the Euler equation (4). The constraints, however, can be easily derived from the relationships specified (or implied) in the Romer paper. By referring below to the equations relevant to their derivation, it is hoped that we can identify, in Section III, what has gone “wrong” with the Romer model.

The production function of the Y sector takes the following Dixit-Stiglitz form:

$$Y(t) = H_Y^\alpha L^\beta \sum_{i=1}^{A(t)} x_i^{1-\alpha-\beta} . \quad (5)$$

The Y sector maximizes the extra profits (applying the symmetry result):

$$\pi_Y = Y - w_H H_Y - w_L L - \zeta x A , \quad (6)$$

where ζ = the rental price of an intermediate good (this equation, though slightly modified, corresponds to the first equation at p. S86 of the Romer paper). The Y sector is perfectly competitive, so that the extra profits are zero:

$$Y = w_H H_Y + w_L L + \zeta x A . \quad (7)$$

The I -sector firms maximize the monopolistic profits—the difference between the rental income from intermediate goods (which are durable) and the cost of the final good input (ηx); the final good input is durable, so that it incurs interest cost only:

$$\pi_I = \zeta x - r \eta x \quad (8)$$

(this equation corresponds to Romer's Equation (5) at p. S86). The present value of the flow of these monopolistic profits must be equal to the purchasing price of each design (P_A), measured in terms of the final good (the *numéraire* of the economy). Since intermediate goods are durable, they yield profits in perpetuity, so that one has the following "arbitrage equation" (Jones, 2002, Ch. 5):

$$rP_A = \pi_I. \quad (9)$$

Thus, one will have

$$\zeta x = rP_A + r\eta x \quad (10)$$

(this equation is equivalent to Romer's Equation (6') at p. S87).

The production technology of the R sector is represented by

$$\dot{A} = \delta H_A A. \quad (11)$$

The R -sector firms maximizes the profits

$$\pi_R = P_A \dot{A} - w_H H_A \quad (12)$$

(this equation is derived from the model specification). From this one gets

$$P_A \dot{A} = w_H H_A \quad (13)$$

(this relationship is referred to at p. S85).

Two points in particular deserve attention. First, as the accounting relationship (7) shows, the output (= the revenue) of the Y sector is distributed as the incomes to the "factors of production" employed in the sector: the wage for human capital, the wage for labor and the rental (interest) on the "capital" of the sector. The "capital" of the Y sector is, *by definition*, the value of the intermediate goods used in the sector, which is calculated as the sum of the quantities of the intermediate goods multiplied by their *prices*. Romer does not refer to the

prices of intermediate goods; however, they are easily derived. From (10) and (2), one gets

$$\zeta xA = r(P_A A + K). \quad (14)$$

Considering that ζ is the rental rate on an intermediate good, that an intermediate good is fully durable (non-depreciating), and that the (uniform) price of an intermediate good (p , measured in terms of the final good) is constant in equilibrium, the relationship between the rental rate and the price of an intermediate good is given as $\zeta = rp$. Thus, the value of the intermediate goods, measured in terms of the final good, is

$$pxA \equiv V = P_A A + K, \quad (15)$$

and the total price of an intermediate good of the amount of x is

$$px = P_A + \eta x. \quad (16)$$

It transpires that the “capital” of the Y sector is more than Romer’s “accounting measure of capital,” contrary to what Romer seems to think (see the first quote from Romer at the beginning of this section). In a model essentially the same as the Romer model, Jones (2005) instead focuses on V , which he calls the “financial wealth” representing the total value of assets the representative consumer possesses. For a later discussion, I draw attention to a salient characteristic of the respective components of V (and therefore also their aggregate). They are entirely independent of the rate of interest: from (13), $P_A A = \delta^{-1} w_H$ (w_H is determined as equal to the marginal product of labor) and K is Romer’s “accounting measure of capital”—the amount of accumulated foregone consumption.

For the problem of optimization by the representative consumer, Romer (implicitly) sets up two budget constraints, with two state variables for the associated Hamiltonian. The first is derived from the combination of (1) on the one hand and (7) and (14) on the other, which gives rise to the equation of motion for K :

$$\dot{K} = w_H H_Y + w_L L + r P_A A + r K - C . \quad (17)$$

The second constraint is the production technology of the R sector, which yields the equation of motion for A .

Alternatively, following Jones (2005), one can construct one budget constraint, with V as the sole state variable. Using equations (7), (10) and (11), and also using (1) and (2), one gets

$$\dot{V} = w_H H + w_L L + r V - C . \quad (18)$$

where $H = H_A + H_Y$. Of course, both cases yield an identical result: the Euler equation (4).

For the problem of optimization by the social planner, the constraint (17) must be appropriately modified. For this, Romer (and also Jones) uses the measure of the “accounting measure of capital”—equation (2)—in order first to reduce the production function of the Y sector (5) to

$$Y = (H_Y A)^\alpha (L A)^\beta K^{1-\alpha-\beta} \eta^{\alpha+\beta-1} ; \quad (19)$$

and then to get the budget constraint

$$\dot{K} = \left[(H_Y A)^\alpha (L A)^\beta K^{1-\alpha-\beta} \eta^{\alpha+\beta-1} \right] - C \quad (20)$$

(see Romer, 1990, pp. S97 and S100).

III. The first problem: a “man-made perpetual machine”

To dig out the (first) problem that the Romer model conceals unwittingly, it is necessary to realize first that the intermediate goods input in the Y sector and the final good input in the I sector are *both* assumed to be *fully durable* inputs. That intermediate goods are durable is explicitly emphasized by Romer. That the final good input in the I sector is durable is confirmed by equation (8), where it incurs only the interest cost and no depreciation cost. I

argue that this seemingly innocent setting³ gives rise to problems, including internal inconsistency, in several ways.

A non-depreciating input is used, by definition, forever. Since the final good input used for the production of intermediate goods is fully durable, it will, once installed for production, be used over and over again, for good, to produce intermediate goods. The other input in the i th intermediate firm is the i th design; this input is also a fully durable input. Since all the inputs are fully durable, the i th type of intermediate goods is produced over and over again, forever, once those inputs are put into production. The point here is that the inputs need to be put into production *only once* in order to repeat production *in perpetuity*. This situation is in compliance with the use of the final output envisaged in (1) and (2): the necessary foregoing of the final output every period (“year”) is only for generating *new* varieties of intermediate goods—no more foregoing is necessary to repeat the production of the already existing intermediate goods. But this situation violates a fundamental law of physics: Man, with a scarce resource, has created a “perpetual machine.” The mere elapse of time will trigger the inputs in the I sector, which were installed once but destined to operate forever, and that as a result there will be, with the mere elapse of time, an increase in the output of intermediate goods. There is, so to speak, “automatic” (and perpetual) increase in the quantity of intermediate goods, once produced. This case should be distinguished from the familiar case of non-depreciating capital, say, in the usual Solovian one-sector model. In the latter case, the stock of capital does not increase unless there is further foregoing of consumption. In contrast, in Romer’s case, the stock of intermediate goods increases automatically even if there is no further foregoing of consumption. This is truly a case of “perpetual machine”—a physical

³ “The analysis is further simplified by assuming that the durables do not depreciate. Adding depreciation would merely add a familiar term to the user cost of capital” (Romer, 1990, p. S82).

impossibility.⁴

There are more serious difficulties, related to *internal logic*. Because the inputs for the i th intermediate goods are installed by given constant quantities (the final good input by ηx_i and the design input by one unit), their “annual” production will be by an “annually” constant amount x_i . Then, the “perpetual machine” aspect leads to the following. At the present time t , after an elapse of time by s_i periods since the i th type of intermediate goods were produced for the first time at time τ_i ($s_i = t - \tau_i + 1 \geq 1$), this type of intermediate goods will have been *accumulated* at the level of

$$s_i x_i = (t - \tau_i + 1)x_i \geq x_i. \quad (21)$$

It will transpire that this situation is in direct conflict with that envisaged by the Dixit-Stiglitz production function (5) of the final good sector. Here the quantity of the i th type of intermediate goods which contribute to the production of the final good at time t is at the constant level of x_i , regardless of how many periods have elapsed since they were produced for the first time. But the “perpetual machine” aspect implies that, at time t , the stock of the

⁴ Romer considers his setting as a convenient simplification, probably having in mind the usual Solovian one-sector model where capital is assumed not to depreciate (see Romer’s statement quoted in the previous footnote). However, there are two critical differences between the Romer model and the Solovian model. The first is the one just referred to in the text. The second is that, in the Solovian model, the “production” of capital by means of the final good is not by an actual process of production: the “foregoing” of consumption is sufficient and necessary; in contrast, an actual process of production, characterized with the use of heterogeneous inputs and with the passage of time, takes place for the capital of the Y sector. This second difference in particular, as will be argued in detail in Section V, leads to the problem of value inherent but put aside in the Romer model.

i th type of intermediate goods which began to be produced s_i (≥ 1) periods earlier must be $s_i x_i$. The production function (5) *underestimates* the quantity of the i th type of intermediate goods which contribute to the production of the final good.⁵ If one accepts Romer's model-setting where all the inputs are durable, then *logic* dictates that the production function in the Y sector be⁶

$$Y(t, \{s_i\}) = H_Y^\alpha L^\beta \sum_{i=1}^{A(t)} (s_i x_i)^{1-\alpha-\beta} . \quad (22)$$

Not only this. Consider the following. Romer's K is the amount of foregone consumption which "evolves according to the rule $\dot{K}(t) = Y(t) - C(t)$ " (Romer 1990, p. S82). Meanwhile, as has been argued just above, the physical units of the i th type of intermediate goods at time t are $s_i x_i$. Then, "[b]ecause it takes η units of foregone consumption to create one unit of any type of durable [intermediate good]" (Romer, 1990, p. S82), the following relationship should have been the case:

$$K = \eta \sum_{i=1}^A s_i x_i . \quad (23)$$

⁵ Romer (1990, p. S86) states that "it is convenient to assume that the capital is putty-putty, so that the firm can ... [convert] units of durables back into general capital, and avoid the interest cost if it decides to supply fewer units.... [T]he demand for durables is stationary in equilibrium, so no disinvestment ever takes place." This statement may apply to individual firms but not to the economy as a whole: the "general capital" in the economy as a whole will at any rate continue to be accumulated in accordance with $\sum s_i x_i$.

⁶ As an aside, the reader will easily check that this (corrected) form of the Dixit-Stiglitz production function does not bring about a symmetric equilibrium: the equilibrium "annual" quantity of the i th type of intermediate goods, x_i , depends on the length of time (s_i) which has elapsed since they were first produced.

K in (23) expresses the amount of the final good accumulatively “embodied” in the intermediate goods.⁷ One will, however, immediately notice serious dissonance between K construed this way and K understood as the amount of the final good accumulatively “foregone,” as is derived from (1). The logical requirement is that the two K ’s be identical (Romer treats them so). The fact is otherwise. Because $\dot{K}(t)$ in (1) is the foregone final good which is used only for producing *new* varieties of intermediate goods, it is affected by time through one route only: the variety of intermediate goods, $A(t)$, as is shown in (2). By contrast, K in (23) represents a quantity that is affected by *more than* $A(t)$: it would increase even if there should be no further increase in the variety of intermediate goods. Consider the case in which there is no change in A . Then, one quantity— K implied in (1)—would not change over time whereas the other quantity— K in (23)—would increase over time. If there are changes in A , the two quantities will grow over time at unequal rates, and the equality relationship between the two K ’s, should it be established for once, will sooner or later break down. The two K ’s cannot be identical. The “perpetual machine” aspect thus leads to internal inconsistency: the two quantities—the amount of the final good accumulatively foregone for producing intermediate goods and the amount of the final good accumulatively “embodied” in the intermediate goods—cannot be identical, though the internal logic requires them to be identical.

One direct casualty of this inconsistency is the budget constraint for the social planner’s optimization problem (20), where the reduced form (19) of the production function is used. K appearing in (19) or (20) is simply the amount of the final good accumulatively “foregone.” However, what matters for the production activity in the Y sector is the number of intermediate goods and, therefore, the amount of the final good relevant to the production

⁷ Romer does not use the term “embodied”; however, as will be made clear, my use of the term is not accidental (see Section V below and footnotes 17 and 18 there).

activity—thus, legitimately to appear in the production function (19)—is one that is accumulatively “embodied” in the intermediate goods. We know from the “perpetual machine” aspect that this amount of “embodied” final good is that given by the right-hand side of relationship (23).

Physics and logic—or “zero tolerance for intellectual sloppiness” (Romer, 2005)—dictate that the model be modified.

IV. A correction midway

Modification directed to the production function (5) makes little economic (and also little physical) sense. A more sensible way is to modify the setting regarding the durability of the inputs.

Suppose that the intermediate goods input in the Y sector is worn away entirely in every “round” of production. But, since the final good input in the I sector is fully durable, the i th type of intermediate goods is newly produced by the amount of x_i every year forever. This situation is in full compliance with the one pictured by the production function (5). Also, there is no need to forego the final output further in order to produce the i th type of intermediate goods, in conformity to the use of the final good envisaged in (1): the final good which is foregone every year is only for producing new varieties of intermediate goods.

If, alternatively, the intermediate goods in the Y sector are durable but the final good input in the I sector depreciates fully in one “round” of production, then ηx_i units of the final good are completely used up to produce x_i intermediate goods of the i th type. Since the foregone output is used only for producing new varieties of intermediate goods, there will be no further production of the i th type intermediate goods. However, because the intermediate goods of the i th type, once produced, will stay in production forever, they will continue to make contributions to production and are entitled a logically valid place as specified in the

production function (5).⁸

The modification made in this paper is to pay respect to Romer's emphasis that intermediate goods used in the Y sector are durable.⁹ Then, to maintain the internal consistency of the model, the final good input in the I sector must be fully nondurable. This implies that the user cost of the final good input must include not only the interest cost but also the *depreciation cost* (at the rate of unity).

But, then—this is a subtle point—a similar reasoning must apply to a design. The i th design is used exclusively for producing the i th type of intermediate goods. The i th type of intermediate goods is produced only for one “round.” Then, *for the producer* of a product which will be produced only once and to which a design is exclusively applied, that design must be as if a *nondurable* good. The cost of using the design having already been paid by the producer, that cost must be entirely recouped in the single “round” in which the corresponding intermediate goods are produced. This requires that the user cost of a design (therefore, the price of an intermediate good) cover not only interest on a design but also the “depreciation” cost of the design. This understanding does not contradict the fact that the i th design will remain in the economy forever; whereas it will no longer contribute to the production of the

⁸ The third alternative, where both the final good input and the intermediate goods input are completely nondurable, is the case of Barro and Sala-i-Martin (2004, Ch. 6).

⁹ The other choice leads to exactly the same result as is reported in the present paper. My choice makes the Romer model closer than before to the Solovian one-sector model with non-depreciating capital, by removing the first difference between the two models mentioned in footnote 4. But the second difference still remains; in fact, it cannot be removed. And it is this difference, as is discussed in detail in Section V, that causes a problem to the Romer model (but not to the Solovain model) in using the proposed measures of capital in the respective models.

i th type of intermediate goods (because there will be no further production of the i th type of intermediate goods), it, as a public good, continues to contribute to the invention of new designs: it is a durable good for the economy as a whole.

The “one-round only” production of the i th intermediate goods will also require the modification of the definition of the gross revenue of the i th firm: this must now be the total *price* of its product, $p_i x_i$, not the rental, $\zeta_i x_i$.¹⁰ The rental will now represent purely the user cost of intermediate goods for the final good producers: they purchase the i th type of intermediate goods at the total price of $p_i x_i$ and recoup this cost in the form of the flow of net revenue over infinite time horizon.

These understandings lead to the following modifications of the net revenue (monopolistic profits) in the I sector and of the “arbitrage equation”:¹¹

$$\pi_I = px - (1+r)\eta x; \quad (8^*)$$

$$\pi_I = (1+r)P_A, \quad (9^*)$$

which together yield the following “price equation” for intermediate goods of the amount of x :¹²

¹⁰ Here I am assuming that the producer of an intermediate good *sells* the good to a Y -sector firm. In an alternative setting in which she *rents* the good (over infinite periods), the “annual” gross revenue should be the rental $\zeta_i x_i$; with the appropriately modified “annual” cost of the final good input, the present value of the flow of “annual” net revenue will be the same as (8*) below.

¹¹ I use the symmetry result in all the modified formulations below; each of the modified relationships carries with it the number corresponding to the one in Sections II but is marked by an asterisk.

¹² The price equation of a good expresses the price of that good as the sum of the all costs

$$px = (1+r)P_A + (1+r)\eta x. \quad (10^*)$$

The total price of a type of intermediate goods of the amount of x is the sum of the user cost of the corresponding design and the user cost of the final good input; both user costs include the depreciation term at the full rate.

No modification is necessary to the remaining sectors: the “price equation” for the final good continues to be given by (7) and for a design by (13). Now, substituting (10*) into (7), one gets

$$Y = w_H H_Y + w_L L + r[(1+r)P_A A + (1+r)K]. \quad (14^*)$$

This shows vividly that the “capital” of the Y sector—which consists of durable intermediate goods and on which interest accrues at the rate of r —is now measured as

$$Z \equiv (1+r)P_A A + (1+r)K. \quad (15^*)$$

One notes outright a difference from the uncorrected Romer model. The “capital” of the Y sector (Z) is more than K ; and, even in contrast with Jones’ “financial wealth” (V), both of its components are not independent of the rate of interest. A more detailed discussion of this point is reserved for Section V.

The counterpart of the budget constraint (17) of the uncorrected model—the equation of motion for K —is

$$\dot{K} = w_H H_A + w_L L + r(1+r)P_A A + r(1+r)K - C. \quad (17^*)$$

The second budget constraint—the equation of motion for A —is the same as in the uncorrected model. Alternatively, in the Jones (2005) way, one can opt to use one budget

incurred in producing it, including interest (rental) accruing to the produced inputs used (in long run equilibrium, the relevant rate of interest is the “normal” rate prevailing uniformly across the economy).

constraint with one state variable, V , as follows:¹³

$$\dot{V} = w_H H + w_L L + r(1+r)V - C. \quad (18^*)$$

Either way, one gets the Euler equation as follows:

$$\frac{\dot{C}}{C} \equiv g = \frac{r(1+r) - \rho}{\sigma}. \quad (3^*)$$

Appendix solves the modified Romer model for the competitive equilibrium. It transpires that the Romer model is robust, except to the extent that $r(1+r)$ replaces r in the solution value of every variable for the decentralized economy of the uncorrected model.

One obvious beneficiary of the above correction is the reduced form of the production function (19) and its use in (20) for the social optimization problem. Now, K appearing in (19) is the amount of the final good accumulatively “foregone” in accordance with (1) and, at the same time, the amount of the final good accumulatively “embodied” in the stock of intermediate goods; the two quantities, which turned out to be divergent from each other in the uncorrected Romer model, are identical as they should be. Thus, equation (20) is the legitimate equation of motion for K for the social optimization problem.

V. The second problem: a “foregone consumption theory of value”

It may seem that the impact of the correction is miniscule, with the correction leaving the robustness of the model intact. However, I contend that my argument up to now points to a fundamental *theoretical* issue at stake in horizontal innovation models in general. All horizontal innovation models have the same—three-sector(R, I, Y sectors)—structure, and this

¹³ However, now, V is not the economically proper measure of the “financial wealth” in reference to which interest is to be calculated. This measure should be Z , instead; see Section

structure is bound to be associated with the problem of *value*.¹⁴ I contend that the current practice in the literature is to (illegitimately) assume this problem away and that the embodiment of such a practice is such measure of capital as Romer’s “accounting measure of capital” or Jones’ “financial wealth,” which is independent of the rate of interest. This practice is, however, *from the theoretical perspective*, not proper because the capital of the Y sector—which must be defined as the value of the intermediate goods used in the sector—cannot be defined, given the three-sector structure of horizontal innovation models, in independence of the rate of interest without violating the fundamental principles of economics.

Let us consider the reduced form of the production function (19). In the modified model, it represents properly the equation of motion for K for the social optimization problem. This is because K now represents identically the quantity of the amount of the final good accumulatively “foregone” and its amount accumulatively “embodied.” But this does not mean that it is also the quantity of “capital” on which the rate of interest is properly defined and calculated. The reduction to (19) has the effect of expressing the production process in the Y sector in terms of three kinds of “factors of production”—human capital (H_Y), labor (L) and the “accounting measure of capital” (K)—with “labor and human capital augmenting technical change” (A) and with α , β and η as technical parameters.¹⁵ However, then, it also

¹⁴ The problem of value arises whenever (i) there are more than one kind of goods in the economy and/or (ii) production takes time. The first situation necessitates a selection of the standard of measurement and the consistent use of it in measuring various quantities in the economy. The literature deals with this situation properly. The second situation invokes the rate of interest: a positive elapse of time involves waiting, and the rate of interest is the reward for waiting. The present section is about the inadequacy, from the theoretical perspective, of the treatment of this second situation in the current literature on horizontal innovation.

¹⁵ Observe Romer’s remark that “[equation (19)] shows that the model behaves just like the

has the effect of possibly making a false impression that the revenue of the Y sector will be distributed in the form of the returns to the three “factors of production”: the wage to human capital ($w_H H_Y$), the wage to labor ($w_L L$) and the rental on the “accounting measure of capital” (ζK).¹⁶ This false impression could be strengthened by the (correct) fact, represented by equation (1), that the same final good output is either consumed or foregone for producing intermediate goods, K being the accumulated quantity of foregone final good. There is a one-to-one relationship between the final good foregone every year (\dot{K}) in (1) and a “factor of production” called the “accounting measure of capital” (K) as the accumulated foregone final good in the production function (19).

The truth is, however, that K is *not* the quantity of capital of the Y sector in reference to which the rate of interest of the sector is to be defined. To be able to talk of the rate of interest on “capital,” this “capital” must be in principle a *single* quantity. Now, the capital of the Y sector in its physical form is the stock of heterogeneous intermediate goods. Heterogeneous goods cannot be summed to yield a single quantity without being first measured into a homogeneous quantity. Indeed, Romer’s “accounting measure of capital” is a result of such a process of homogenization. Each unit of an intermediate good uses η units of the final good; that is, an intermediate good “embodies” η units of the final good.¹⁷ Romer must have

neoclassical model with labor and human capital augmenting technical change. In particular, it exhibits the usual diminishing returns to capital accumulation” (Romer, 1990, p. S89).

¹⁶ Because K was already measured in terms of the *numéraire* (the final good) and because intermediate goods were fully durable, one would have $\zeta = r$.

¹⁷ The reader will soon notice a close similarity between Romer’s process of obtaining K (if construed as the measure of “capital” of the Y sector) and the (in)famous (and eventually refuted) measurement in Marxist economics of “constant capital” solely in terms of

reasoned that the quantities of different types of intermediate goods can be compared with each other (and thus added up) in terms of this “embodied” final good. Summed over A varieties of intermediate goods, each of the amount of x , the total quantity of the final good “embodied” in the stock of intermediate goods is $\eta x A$, and this is his “accounting measure of capital.” However, due to the structure of the Romer model (and, by extension, of all horizontal innovation models), there is an inherent gap between the amount of the final good accumulatively foregone—which is described by the first quote from Romer in Section II—and the quantity of “capital” existing in the Y sector—which the second quote implicitly describes, wrongly, as equivalent to the former amount.

To see this, the reader is urged to realize (or recall from old controversies in capital theory; e.g., Bliss, Cohen and Harcourt, 2005) that the measure of capital of the Y sector relevant to the calculation of the rate of interest prevailing in that sector must be the measure obtained in reference to the *prices* of the intermediate goods used in the Y sector (these prices being relative to the chosen standard of measurement of the economy—in the current case, the final good). This is because, obviously, the intermediate goods are sold and purchased in accordance with their prices and because the value of intermediate goods calculated in reference to their prices is what costs the producers of the Y sector when they purchase them as the “capital” for their production activity. The measure of capital obtained this way will diverge from the “accounting measure of capital” insofar as the prices of the intermediate goods are not proportional to the quantities of consumption “foregone” for (and “embodied” in) the respective goods. It will transpire that non-proportionality is the general case.¹⁸

In equilibrium, the price of a good must be no more, and no less, than the sum of all the “embodied” labor.

¹⁸ If proportionality holds, we shall be talking of the “foregone consumption theory of value”—in homage to the embodied labor theory of value.

costs incurred in producing that good, including interest on any produced inputs used in production (in long run equilibrium, the rate of interest must be uniform across the produced inputs and across the sector). Now, the production of a type of intermediate goods requires the input of the final good (η units for one unit of an intermediate good) and, *in addition*, the input of a design. The price of an intermediate good must include the user costs of both of these inputs; it follows that the aggregate value of intermediate goods (the “capital”) must also reflect the user costs of both of these inputs. However, K is the value of intermediate goods which reflects the cost of the final good input only. The quantity of “capital” of the Y sector in reference to which the rate of interest of the sector is to be defined diverges from the accumulated final goods that have been foregone for producing them. The price of an intermediate good is not proportional to the quantity of the final good foregone for that good.

The matter does not stop here (and this point applies *mutatis mutandis* to all currently available horizontal innovation models). I have argued that, in the model-consistently modified model, the quantity of capital of the Y sector is

$$Z = (1+r)P_A A + (1+r)K . \quad (15^*)$$

Interest is the reward to waiting. Because the providers of a design and of the final good input in the I sector have to wait for one period until they recoup the costs that were incurred in producing and providing them, this waiting must be rewarded by interest: the rate of interest r appearing in (15*) represents this aspect (we are concerned with long run equilibrium, so that the rate of interest is uniform across the inputs). Then, the interest generated in the Y sector is

$$\zeta x_A = r p x_A = r Z = r [(1+r)P_A A + (1+r)K] . \quad (24)$$

The powers of the rate of interest in (24) reflect the number of periods which the respective inputs used in producing the intermediate goods, which constitute the “capital” of the Y sector, have to go through until the final good is produced: both the designs and the foregone final

good go through two periods since the moment when they were first put into use for production in the I sector.

This observed, it transpires that the quantity of capital that Jones (2005) uses (“financial wealth,” V) is also an incorrect measure of capital even within the all-durable-inputs setting of the original Romer model. Two periods pass from the moment when both designs and foregone final good are put into use in the I sector to the moment when the final good is produced with the aid of intermediate goods (which are produced by means of designs and foregone final output). These inputs do not depreciate. However, time flows regardless of the durability of inputs; therefore, waiting is involved and this waiting must be rewarded in terms of interest. The user costs of a design and of the final good input respectively in the I sector (therefore the price of a type of intermediate goods of the amount of x) will, as they are fully durable, include only the interest cost of a design and the interest cost of the final good input. The provider of an intermediate good charges this price every period, in perpetuity:

$$px = rP_A + r\eta x . \quad (25)$$

The present value of the perpetual flow of this price, obtained as the revenue every period, equals the total purchasing cost of the durable inputs: $P_A + \eta x$. Observe how equation (25) is different from what I have treated as the implied price equation of the Romer model, equation (16), on which Jones’ “financial wealth” relies. Now, within the all-durable-inputs setting, the “correct” value of the intermediate goods used in the Y sector—the “capital” of the Y sector—must reflect my argument in Section III regarding the physical quantity of the intermediate goods; thus, it should have been

$$V' = (rP_A + r\eta x) \sum_{i=1}^{A(t)} s_i x_i . \quad (26)$$

I draw the reader’s attention to the common characteristic of the measures of capital, given in (15*) and (26), in contrast with the ones that have been used in the literature—

Romer's "accounting measure of capital" (K) and Jones' "financial wealth" (V). The latter ones are independent of the rate of interest, whilst the former ones involve the rate of interest. The difference lies in whether the measures reflect the passage of time that an input in production is subject to from the moment of its being put into use to the moment of the final good being produced. This passage of time is ignored in the latter measures. But the passage of time is an essential aspect of real production, and this is all the more the case for horizontal innovation models. In the I sector, inputs are installed to produce intermediate goods; one period of production passes in the transformation of the inputs into the output; from the perspective of the I -sector firms, the price of these intermediate goods must include interest on the inputs because they have to wait for one period until they can recoup the costs of the input by selling (or renting) the output. The stock of these intermediate goods is the capital of the Y sector, in its physical form; it follows that the value of capital of the Y sector, which is a single quantity obtained by aggregating the heterogeneous intermediate goods with the aid of their prices, should involve interest in it.

Romer states: "In parallel with the usual one-sector model and in conformity with national income accounting conventions, it is useful to define an accounting measure of total capital K as cumulative foregone output" (p. S82). The problem is that a horizontal innovation model is not a one-sector model, but a three-sector model. His "accounting measure of capital"—and, to a lesser extent, its variants such as Jones' "financial wealth"—is an ingenious device which simplifies the formulation of the dynamic optimization problem, but it is at the same time the product of an illegitimate maneuver of treating a multi-sector economy as a one-sector one.

VI. Conclusion

The Romer (1990) model of horizontal innovation treats three quantities as identical, all

expressed as K , uniformly under the name of the “accounting measure of capital”: the amount of the final good accumulatively “foregone” for producing intermediate goods, the amount of the final good accumulatively “embodied” in the intermediate goods, and part of the quantity of “capital” in reference to which the rate of interest is to be defined. Logic (and also physics) requires the first and the second quantities to be identical. But Romer’s model-setting in which all the inputs in the economy are durable (the “perpetual machine” aspect) implies that they diverge from each other—hence, internal inconsistency. The correction in the present paper has restored the required identity.

Meanwhile, the structure of the Romer model requires, by the economic logic, the second (and, thus, the first) and the third quantities to differ from each other in general. What is involved here is the problem of value. The Romer model (and, by extension, all other horizontal innovation models), with the three-sector structure where “commodities are produced by means of commodities” (Sraffa, 1960), is inevitably subject to the problem of value; in particular, the problem of interest and the measurement of capital. The elapse of time involved in the sequential process of production necessitates the compounding of interest and the involvement of interest in the measure of capital in the final good sector. It is this involvement of time and, thus, of interest which requires that Romer’s third K (part of the quantity of “capital”) should in general diverge from his second K (the quantity of consumption “embodied” in the intermediate goods). By treating them identical (the “foregone consumption theory of value”), Romer has assumed away an aspect that is inherent to his model—the problem of value, interest and capital.

Romer’s “accounting measure of capital” (and its variants in the literature) is an embodiment of a practice which is not proper from the theoretical perspective. While being an ingenious device that makes simpler the mathematical formulation of the model, the measure is a sleight of hand that violates fundamental principles in value and capital theory. A

fundamental principle in price theory is that the price of a good includes all the costs incurred in its production, including the cost of time. A fundamental principle in capital theory is that the rate of return on the capital whose physical form is the stock of heterogeneous goods, must be calculated on the *value* measure of capital which is obtained in reference to the prices of the physical components of the capital. The prices used in obtaining Romer’s “accounting measure of capital” (and its variants) ignore the cost of time, which should have revealed itself as the interest factor at a compound rate. The measures of “capital” (or “assets”) currently used in the horizontal innovation literature—as quantities independent of the rate of interest—are the result of brushing away the problem of value inherent in the three-sector structure of the models. However, the problem of value, once given birth, cannot be simply brushed away but is bound to make a conspicuous appearance somewhere sometime. Romer’s own “zero tolerance for intellectual sloppiness” will duly recognize such a birth and appearance.

Appendix

Solving the model-consistently modified Romer model

The total human capital and labor in the economy are given, respectively, at the levels of H and L ; human capital is divided into the research sector by the amount of H_A , and into the final good sector by H_Y , and labor is used entirely in the final good sector:

$$H_A + H_Y = H . \tag{A1}$$

The final good sector utilizes the following Dixit-Stiglitz technology:

$$Y = H_Y^\alpha L^\beta \sum_{i=1}^A x_i^{1-\alpha-\beta} . \tag{A2}$$

Profit maximization in this sector yields, first, the demand function for each type of

intermediate goods

$$\zeta_i = (1 - \alpha - \beta)H_Y^\alpha L^\beta x_i^{-\alpha-\beta}, \quad (\text{A3})$$

where ζ_i is the rental rate on the i th type of intermediate goods; and, second, the wage rates paid to human capital and labor, respectively

$$w_H = \frac{\partial Y}{\partial H_A} = \alpha H_Y^{\alpha-1} L^\beta \sum_{i=1}^A x_i^{1-\alpha-\beta}; \quad (\text{A4})$$

$$w_L = \frac{\partial Y}{\partial L} = \beta H_Y^\alpha L^{\beta-1} \sum_{i=1}^A x_i^{1-\alpha-\beta}. \quad (\text{A5})$$

Because the intermediate goods input is fully durable, the following must be the case:

$$\zeta_i = r p_i, \quad (\text{A6})$$

where p_i is the price of the i th type of intermediate goods.

The i th intermediate good firm utilizes the i th design and ηx_i units of the final good to produce x_i units of the i th intermediate good. Arbitrage regarding the investment on designs yields the following “price equation” (noting that the final good input is exhausted in one “round” of production and that a design is accordingly taken as if a nondurable input to the producer of an intermediate good):

$$p_i x_i = (1+r)P_A + (1+r)\eta x_i. \quad (\text{A7})$$

Profit maximization by each firm in the intermediate goods sector results in the following monopoly pricing rule (recall that the gross revenue to a firm is now the total sale of, not the total rental on, the intermediate goods by the firm):

$$p_i = p = (1 - \alpha - \beta)^{-1}(1+r)\eta. \quad (\text{A8})$$

(By virtue of (A8), we can henceforth drop the subscript i from all variables.)

The research sector utilizes H_A of human capital and the stock of previous designs (A , which are public goods) in such a way that

$$\dot{A} = \delta H_A A. \quad (\text{A9})$$

Profit maximization in this sector determines w_H as the marginal value product of human capital:

$$w_H = P_A \frac{\partial \dot{A}}{\partial H_A} = P_A \delta A, \quad (\text{A10})$$

which is the “price equation” of the research sector.

The representative household is faced with the problem of maximizing

$$\int_{t=0}^{\infty} \frac{C(t)^{1-\sigma}}{1-\sigma} e^{-\rho t} dt, \text{ subject to} \quad (\text{A11})$$

$$\dot{K} = w_H H_Y + w_L L + r(1+r)P_A A + r(1+r)K - C; \quad (\text{A12})$$

$$\dot{A} = \delta H_A A; \quad (\text{A13})$$

or, instead of (A12) and (A13),

$$P_A \dot{A} + \dot{K} = w_H H + w_L L + r(1+r)P_A A + r(1+r)K - C. \quad (\text{A14})$$

Finally, we are concerned with balanced growth equilibrium; thus,

$$\frac{\dot{C}}{C} = \frac{\dot{A}}{A} (= g). \quad (\text{A15})$$

The system consists of the relationships (A1) to (A13) and (A15). One notices that this system is different from the original Romer model by additionally having (A6), (A7) and (A8), which are instrumental to obtain (A12) or (A14).

A series of calculations produces the following:

$$P_A = \left[\frac{\alpha + \beta}{r(1+r)} \right] \left[(1 - \alpha - \beta) H_Y^\alpha L^\beta x^{1-\alpha-\beta} \right]. \quad (\text{A16})$$

$$H_Y = \delta^{-1} \Lambda r(1+r), \quad \Lambda \equiv \frac{\alpha}{(1 - \alpha - \beta)(\alpha + \beta)}; \quad (\text{A17})$$

$$\frac{\dot{A}}{A} = \delta H - \Lambda r(1+r). \quad (\text{A18})$$

$$\frac{\dot{C}}{C} = \frac{r(1+r) - \rho}{\sigma}. \quad (\text{A19})$$

The difference from the uncorrected Romer model is that $r(1+r)$ replaces r in every counterpart of the Romer model. This implies that all the variables, except r , have the same solution values as in the uncorrected Romer model. Combining (A18) and (A19), one gets

$$r^2 + r - \Omega = 0, \quad \text{with } \Omega \equiv \frac{\rho + \sigma \delta H}{1 + \sigma \Lambda}. \quad (\text{A20})$$

Since $\Omega > 0$, (A20) gives a unique positive rate of interest:

$$r = \frac{-1 + (1 + 4\Omega)^{\frac{1}{2}}}{2}. \quad (\text{A21})$$

The uncorrected Romer model *overestimates* the equilibrium rate of interest, by the interest factor of $(1+r)$.

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