

A Model of Endogenous Agricultural Commodity Price Fluctuations with Storage

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Abstract

A debate has been raging for centuries regarding the effects of inter-annual storage on commodity prices, considered by most as a stabilizer, while others place it at the core of the explanation of intriguing features of commodity price series, such as skewed distributions. Most of these studies have been developed in the context of the theory of competitive storage where random shocks affect supply or demand. Recently, the endogenous chaotic behavior of markets has become another possible hypothesis regarding the origin of commodity price fluctuations. We develop a nonlinear cobweb model with intra and inter-annual storage, risk averse agents and adaptive expectations. Like the theory of competitive storage, this nonlinear cobweb model with storage can reproduce some of the stylized facts of agricultural commodity prices (autocorrelation of first rank, low kurtosis and skewness). In addition, we assess the consequences of inter-annual storage in such a different context. In a nonlinear cobweb model, inter-annual storage is found to reduce price variability.

Keywords: Agricultural Prices; Nonlinear Cobweb Model; Endogenous Fluctuations; Storage

JEL classification: Q11, E39, D84

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1. Introduction

It is well established that agricultural price fluctuations have negative effects on welfare, in particular on children's health (Jensen 2001), food security and growth (Myers 2006) in developing countries. However, there has been much debate around the food policies which can be used to reduce fluctuations. The sharp increase in the prices of agricultural commodities in 2007/2008 has led to a renewed interest in agricultural price fluctuations and how they can be reduced. The design of a policy response to agricultural price fluctuations requires understanding how fluctuations arise in the first place.

Two main types of explanations have been proposed. One is exogenous (Cafiero and Wright 2006; Deaton and Laroque 2003, 1992): weather shocks or any other factor outside the economic environment perturbs supply. The other explanation is endogenous (e.g., Ezekiel 1938): erroneous expectations lead to over or under supply. In both cases, an inelastic demand magnifies imbalances and creates large and detrimental price fluctuations. In the exogenous case, all methods based on the law of large numbers (e.g., insurance schemes, stockpiling, futures markets, widening markets) are efficient to alleviate price volatility. Under the endogenous explanation, on the contrary, the most beneficial strategies are the old recipes of the 1940s such as production quotas and national supply management.

None of these theory are satisfactory in terms of their ability to mimic reality without *ad hoc* assumptions. In particular, time series of agricultural commodity prices typically

exhibit small kurtosis, positive autocorrelation and positive skewness (occasional spikes). Such characteristics are not plausible for series with random shocks, nor are they encountered with simple endogenous fluctuations models such as those by Day and Hanson (1991), Finkenstadt and Kuhnert (1992), Boussard (1996), Hommes (1998), Athanasiou, Karafyllis and Kotsios (2008) .

In the “exogenous fluctuation “ theory context, however, storage has been the subject to much attention for two reasons : first, if fluctuations are of random origin, and liable to the law of large numbers, at least throughout time if not across space, then storage could be a solution to stabilize markets and avoid large fluctuations. Secondly, storage creates a link between two successive periods of time, which makes it a candidate for explaining the existence of serial auto correlation.

For most non economists, the “speculator” is the bad guy whose storing activity artificially creates the scarcity which will allow him to get enormous benefits. On the contrary, the theory of competitive storage (Cafiero and Wright 2006; Wright and Williams 1982; 1984) insists on the fact that, to make money, the inventory holder must buy at low, and sell high price. Such an activity is essentially stabilizing, by increasing demand when prices have to be increased, and supply when they should be lowered. Thus, in face of random shocks affecting supply, storage must be stabilizing.

Even so, famous papers by Deaton and Laroque (D&L thereafter) (1992, 1996) cast doubts on this conclusion : assuming production is subject to random shocks that are identically and independently distributed, the inventory holder knowing the present inventory level, they show that price increases must be expected if the inventory level is

low, and vice versa. The storing strategy of the inventory holder follows immediately. Tacking account that, in addition, the stock must be non negative, and less than a maximum storage capacity, the ensuing prices time series, (as resulting of market clearing conditions) presents remarkable similarities with actually observed commodity prices time series in terms of the first moments of the distribution. The most striking conclusion of this model is that storage can transform a stationary i.i.d. Gaussian exogenous process into a much more complicated time series, with positive skewness, low kurtosis, and auto-correlation. As a consequence, and contrary to the common economic wisdom, storage may be somewhat destabilizing.

The strength of the D&L model is its parsimony in term of assumptions : operators are risk neutral, their price expectations are rational in the sense that they are free of any errors. Yet, like in other competitive storage models, it relies upon an exogenous shock as the basic source of supply and price variability. This is certainly in line with the main stream in agricultural economics, but it is still an assumption that cannot be taken for granted.

In the endogenous explanation, fluctuations are endogenous to agricultural markets and result from the behavior of agents or characteristics of the market (e.g., Voituriez 2001; Chavas and Holt 1993). The cobweb model, originally popularized by Ezekiel (1938), is often used to illustrate this explanation: fluctuations arise primarily due to erroneous expectations that lead to over or under supply. The original cobweb model leads to three simple price trajectories (converging, diverging and periodic) which (except for the “converging” one) are simply not realistic: a periodic motion is impossible, since at least

some operators would notice the periodicity, organize production in order to take advantage of it, and, in doing so, break it. The diverging situation, implying negative prices and quantities after a while, is not possible either. However, the cobweb model has been refined over the past three decades. While keeping the two basic ingredients of the diverging Ezekiel's model (the role of imperfect expectations, and the genuine instability of the "repelling" fixed equilibrium point), by adding mechanisms which call back the system toward equilibrium whenever it is "far" from it, such authors as Day and Hanson (1991) or Hommes (1998) obtained "chaotic" price trajectories that are irregular and thus more realistic than those of the original cobweb model. As explained in Werndl (2009), "chaotic systems are deterministic systems showing *irregular*, or even *random*, behavior and sensitive dependence to initial conditions", which "means that small errors in initial conditions lead to totally different solutions" (emphasis in original).

Chaotic cobweb models of agricultural markets (Athanasίου, Karafyllis and Kotsios 2008; Boussard 1996; Finkenstadt and Kuhbier 1992) have been built along these lines, and exhibit the same apparently random pattern of price fluctuation. Yet, they lead to a dynamic that often exhibits negative skewness and low first order autocorrelation (e.g., Brock, Dindo and Hommes 2007; Hommes 1998) and thus greatly differs from that of actual agricultural prices. As mentioned above, time series of agricultural commodity prices typically exhibit small kurtosis, positive autocorrelation and positive skewness (occasional spikes). How do the two theories above fare when they are put to an empirical test? So far, only the theory of competitive storage has been able to reproduce these stylized facts.

This paper seeks to investigate price dynamics in a nonlinear cobweb model with private storage. It attempts to contribute to the existing literature in three ways. First, with seasonality and storage, our model incorporates standard features of agricultural markets and is more realistic than past cobweb models. Second, we test whether our nonlinear cobweb model reproduces the stylized facts of agricultural price series. Third, we investigate the effect of private inter-annual storage on prices and production.

The rest of this paper is organized as follows. Section 2 provides a motivation for the model of endogenous price fluctuations with storage, seasonality, adaptive expectations and risk aversion, and Section 3 presents the model. Section 4 includes simulation results and an empirical test of the model. Section 5 concludes.

2. Background

We present below four main features of the nonlinear cobweb model developed in this paper. Compared to earlier cobweb models, our model is novel in that it incorporates seasonality and private storage. Like some earlier cobweb models, we use assumptions of risk aversion and adaptive expectations. We argue that these assumptions are as plausible as those of risk neutrality and rational expectations of the theory of competitive storage.

2.1. Feature 1: Seasonality and Private Storage

We extend the cobweb model by including seasonality, an important feature for annual crops. We study the case of a crop year with two seasons: a season when the crop is harvested, “summer” or season 1, and a season when the crop is planted, hereafter, “winter” or season 2. Private storage (i.e. storage by the farmer or the storage firm)

within and between crop years is made possible to allow for consumption throughout the year. The theory of competitive storage has already developed models including intra and inter-annual storage (e.g., Lowry et al 1987). However, prior Cobweb models, including recent chaotic ones, do not incorporate private storage. In the case of seasonal nonperishable crops, private storage though becomes a technical necessity since production takes place only at certain times and consumption occurs all the time.

In addition, storage has been at the center of the theoretical understanding of the dynamics of commodity markets as part of the theory of competitive storage. The now famous papers by Deaton and Laroque (1992, 1996: D&L, for short, in what follows) stand as the most impressive achievement in this direction, showing that storage can indeed take place within a stationary random production setting and be somewhat destabilizing. The reason for these conclusions is that because when stocks are high (low), and perturbations stationary, the probability of a low (high) price during the next period is increased. Thus, even rational a risk neutral speculator is induced to sell (buy) in this context. At the same time, the total storage capacity places upper and lower bounds to the size of the current stock, thus creating non linear relations between the variables of the model, and accounting for sudden large jumps at irregular intervals in the price series as they are observed in actual price series. Results from the theory of competitive storage thus suggest that storage plays a central role in explaining so far the dynamics of commodity prices. Does storage help model commodity price dynamics in an entirely different theoretical framework, i.e. a cobweb model where fluctuations are entirely endogenous? We attempt to answer this question by developing a cobweb model with storage and testing it empirically.

2.2. Feature 2: Segmented Markets

We model an agricultural commodity market that is segmented into a local market where farmers sell the commodity to storage firms and a central market where storage firms sell the commodity to consumers. This scenario is especially relevant for developing economies where production and consumption are geographically dispersed over wide areas and where transport infrastructure is limited. It is also plausible for some regions of developed countries (Bobenrieth, Bobenrieth and Wright 2006).

2.3. Feature 3: Risk Aversion

Planting and storage decisions are made under uncertainty, attitudes toward risk hence become an essential part of the model. Boussard (1996) found that adding risk aversion to an otherwise linear cobweb model can lead to a chaotic dynamics for prices. We start from the model of Boussard (1996) with risk averse agents, to which we add seasonality and private storage, and nonlinear supply and demand curves¹. Thus our analysis differs from the theory of competitive storage, where agents are risk neutral. It is important to note that attitudes toward risk are notoriously difficult to model and measure. We use the mean-standard deviation specification of preferences, which can be justified as a Taylor's expansion of order two of the mean of a concave utility function (Newbery and Stiglitz, 1981). Agents are assumed to be risk averse, which is debatable. The possibility of farmers being risk lover has been considered (Binswanger 1980), and, over the last two

¹ Like Finkelstadt and Khubier but unlike Boussard (1996), we use a nonlinear demand curve and adaptive expectations. However, in another version of the model (Mitra 2001) we use, like Boussard (1996), a linear demand curve and naïve expectations and reach results that are qualitatively similar.

decades, it has been shown that there are yet other possible behaviors with respect to risk such as prudence (Kimball, 1990). In the context of agricultural commodity markets, where large shares of the revenues of farmers and middle men are at stake in planting and storage decisions, our assumption of risk aversion appears to be no less plausible as that of risk neutrality of the theory of competitive storage. Finally, whatever the representation of attitudes toward risk, a difficulty arises while aggregating heterogeneous decision makers: for instance, some farmers could be risk averse, and others risk lovers. In this paper, we stick to the crudest representation with a representative farmer and a representative inventory holder, assuming for each that increasing mean income is "good", and increasing variance is "bad". We do not seek a fine tuning of the model with heterogeneous agents, which could be the subject of further research.

2.4. Feature 4: Expectations

The theory of competitive storage assumes, some times without any defence that agents have rational expectations. Of course, the idea that decision makers process the available information rationally, although perhaps debatable, stays within the tradition of most economic representations of the world, and its relevance is hard to deny. At the same time, the assumption along which they possess beforehand a perfect knowledge of equilibrium prices is probably not tenable, except in classroom models, and for purely theoretical purposes. In a model like D&L's, with production engendered by a zero mean stationary process, it is possible to assume that decision makers know the probability distribution of shocks, (and therefore, of prices), conditional to the current volume of the

stock (if known at decision time, which is at least discussible). This is a way of representing rational expectation based on abundant yet incomplete information (next year's actual price is not known with certainty).

Here, agents have less information, since the probability distribution of prices is not known. Instead, we rely upon the expectation scheme that was deemed the most natural by the authors who first attempted to represent the effect of prices upon production, the adaptive (Nerlovian) expectations of future prices. In this framework, agents only use recent past data to form their price expectations and make systematic forecast errors, which lead to over or under supply. These errors are what lead to endogenous fluctuations in the cobweb model. Given the long tradition of using rational expectations in commodity price models brought about by the theory of competitive storage, at first sight, this assumption may appear implausible. However, recent research on bounded rationality suggests that individuals may form expectations using simple rules of thumb given the cost of collecting and using information (Hommes et al 2007; Chavas, 2000)². While settling the debate between rational and other forms of expectations is beyond the scope of this paper, recent evidence points out that the use of adaptive or other boundedly rational expectation scheme is plausible.

Finally, agents in the model also anticipate the level of price variability. As in Boussard (1996), we use a naïve expectation scheme for the variance, that is the squared difference of the observed price and the expected price for the last season³.

² See also Conlisk (1996) for a review of the evidence on bounded rationality.

³ It is important to note that there are several possibilities for such a variance expectation: the last observed price might be the price observed during the last season, or during the same season last year. The expected

3. A Nonlinear Cobweb Model with Private Storage

The agricultural commodity under consideration is harvested annually, and consumed during two seasons: summer ($i=1$) and winter ($i=2$). The representative farmer does not have the capacity to store, but the representative storage firm (also called inventory holder thereafter) does in both seasons. The storage decision is made by the storage firm in both seasons, while the production decision is made by a representative farmer in winter. The quantity planted by the farmer in winter is available for sale to the storage firm during the summer of the following year.

3.1 Step 1: Modeling segmented markets and a nonlinear demand

Let $p_{t,i}^c$ be the commodity's price of the commodity during season i of year t . How much is produced in summer $q_{t,1}$ is decided during the previous winter. There is no production in winter: $q_{t,2}=0$. In both seasons, $i \in \{1,2\}$, the price is determined by the inverse demand function:

$$p_{t,i}^c = \beta_i (C_{t,i})^{-\alpha_i} \quad (1)$$

where $C_{t,i}$ is the quantity consumed in year t season i , and α_i and β_i are constant scalar coefficients, with $\alpha_i \geq 0, \beta_i > 0$.

$C_{t,i}$, $q_{t,i}$ and $s_{t,i}$, the volume of the inventory decided during season i of year t , are related by the recursive equation:

$$s_{t,i} = q_{t,i-1} + s_{t,i-1} - C_{t,i} \quad (2)$$

price might be derived from a naïve or a Nerlovian expectation scheme. We explored these different possibilities, without noticing significant changes in the outcome: of course, especially when the outcome is chaotic, each of these assumption results in a different series, but the dynamics of the model remains unchanged.

During summer, production is sold by the farmer to the inventory holder at local market price $p_{t,1}^l$, which is assumed to be a linear function of the central market price $p_{t,1}^c$:

$$p_{t,1}^l = p_{t,1}^c - T \quad (3)$$

with $T > 0$ standing for transportation and handling costs. During winter, no exchange takes place on the local market between the farmer and the inventory holder.

3.3 Step 2 : the storage decision

For the storage decision, the question arises as to how the inventory holder forms price expectations. Because the inventory holder operates in the market during both seasons, it is logical that he/she is aware of the seasonality of the market. We assume that the inventory holder has a Nerlovian expectation scheme that takes into account the market seasonality as follows. In season 1 of year t , the price expectation for the next period is:

$$\hat{p}_{t,2}^c = \hat{p}_{t-1,2}^c + \gamma_s (\hat{p}_{t-1,2}^c - p_{t-1,2}^c) \quad (4)$$

In season 2 of year t , the price expectation for the next period is:

$$\hat{p}_{t+1,1}^c = \hat{p}_{t,1}^c + \gamma_s (\hat{p}_{t,1}^c - p_{t,1}^c) \quad (5)$$

In (4) and (5), the hat over a variable indicates an expectation and λ_s is the inventory holder's expectation correction coefficient. This way, during the summer (winter), the inventory holder remembers the price level last winter (summer) to make expectations regarding next winter (summer). This is certainly better than to start from this summer (winter) to guess the next winter (summer) price. In addition, the risk averse inventory holder also forms an expectation regarding the future variance of prices. We use the same scheme as for the producer above, that is:

$$\hat{\sigma}_{t,i+1}^2 = \frac{1}{4}(\hat{p}_{t-1,i+1}^c - p_{t-1,i+1}^c)^2 \quad (6)$$

In the decision of how much to store, the inventory holder maximizes expected utility from sales revenues net of storage costs during the next period. The first order condition for utility maximization is:

$$(\hat{p}_{t,i+1}^c - 2A_s \hat{\sigma}_{t,i+1}^2 s_{t,i})/(1+r) = p_{t,i}^c + c \quad (7)$$

where A_s is the inventory holder's absolute risk aversion coefficient, r the inter-seasonal rate of interest, c is the cost of storage between two seasons, and $s_{t,i}$ is the size of the stock during season i of year t . The left hand side of equation (7) is the discounted value of the certainty equivalent of the expected price, while the right side of equation is the cost incurred to store a unit quantity of the commodity,. The current price $p_{t,i}^c$ of season i plays a role in equation (7). Indeed, $p_{t,i}^c$ is not independent of the storage decision, since the demand for storage competes with the demand for immediate consumption. In season i , the decision to store is simultaneous with the decision to sell (or consume) and the price $p_{t,i}^c$ is a market clearing price, resulting from the arbitrage between storing or selling.

For that reason, according to (1), given $q_{t,1}$, the harvest available in summer (season 1) of year t , and the stock carried over from the previous year $s_{t-1,2}$, $p_{t,1}^c$ is given by:

$$p_{t,1}^c = \beta_1(q_{t,1} + s_{t-1,2} - s_{t,1})^{-\alpha_1}, \quad (8)$$

In winter (season 2) of year t , when there is no harvest, the price is given by:

$$p_{t,2}^c = \beta_2(s_{t,1} - s_{t,2})^{-\alpha_2} \quad (9)$$

subject to $(0 \leq s_{t,i} \leq S_{\max})$ where S_{\max} is the maximum storage capacity.

3.2 Step 3: the production decision

In the winter (season 2) of year t , the farmer makes the production decision for the following summer (season 1 of year $t+1$). The producer maximizes profit, by equating expected price with a non linear marginal nonlinear marginal cost, $K_{t+1,1}$ given by:

$$K_{t+1,1} = bq_{t+1,1} \quad (10)$$

The producer's expectation regarding the commodity price follows the standard Nerlovian model:

$$\hat{p}_{t+1,1}^l = \hat{p}_{t,1}^l - \lambda_p (\hat{p}_{t,1}^l - p_{t,1}^l) \quad (11)$$

where λ_p is the producer's expectation correction coefficient, It is to be noticed that the producer's expectations are based on the previous summer price. There is no exchange between producer and inventory holder during the winter, and hence no local winter price then, and the producer does not know the winter price on the central market . At the same time, the risk averse farmer's expectation of the price variance is given by:

$$\hat{\sigma}_{t+1,1}^2 = \frac{1}{4} (\hat{p}_{t,1}^l - p_{t,1}^l)^2 \quad (12)$$

Then, the producer decides how much to produce by equating the marginal cost with the certainty equivalent of the expected price, solving for $q_{t+1,1}$ in the following equation:

$$bq_{t+1,1} = \hat{p}_{t+1,1}^l - A_p \hat{\sigma}_{t+1,1}^2 q_{t+1,1} \quad (13)$$

The set of equation from (4) to (13) fully determine the variables $p_{t,i}^c$, $p_{t,1}^l$, $s_{t,i}$, and $q_{t+1,1}$.

3. Numerical Simulations

Although simple, the above model is too complicated to allow for a purely analytical treatment⁴. We therefore proceed with simulations with plausible values for the key parameters of the model developed in section 2.

We vary the values of the parameters which are expected to influence the dynamics of the model. Based on prior research on the cobweb model, this is the case for the supply and demand elasticities (Ezekiel 1938; Boussard 1996), the coefficient of expectation correction (Hommes 1994) and the coefficient of absolute risk aversion (Boussard 1996). Using the literature on the theory of competitive storage (Lowry et al (1987)), the cost of storage and the storage capacity are also important parameters in the model so we vary these parameters.

3.1 Calibration

For the demand parameters, we have kept the price elasticity of demand ($\eta_D=1/\alpha_i$) below 1 given that the demand for food commodities is known to be inelastic. For other parameters, in the absence of conclusive empirical measures, we have picked parameter value ranges that are deemed plausible and are given in table 1. It should be noted that the farmer was assumed to be more risk averse than the storage firm, which seems plausible

⁴ We cannot derive from the model above a difference equation that drives price movements with price as the only endogenous variable as has been done in Boussard (1996) and Finkelstadt and Khubier (1992).

on the basis that a storage firm is expected to be wealthier than a farmer (Newbery and Stiglitz (1981; p. 164)).

Finally, the constants β and b in the inverse demand and marginal cost functions have been calibrated in such a way as to maintain prices and quantities within reasonable limits, that is between 0 and 10. One could argue that such a constraint imposes an artificial bound on a key variable of the model. Yet, since the consumer's income is limited, there is obviously an upper limit for prices, precluding a specification of the demand curve which would allow prices growing to infinity. For β (and prices), this is done by a small shift of the demand curve through parameter ε . The inverse demand function is now $p = \beta(q + \varepsilon)^\alpha$. Then, if $q = 0$, $p = \beta\varepsilon^\alpha$. ε is chosen so as to have $\varepsilon^\alpha = 10^5$.

For b (and quantities), the problem is to have $\left(\frac{p}{b}\right)^\alpha \leq 10$ when p reaches its maximum value, 10: this is done by taking $b = 10^{(1-\alpha)}$.

3.2 Price Dynamics

The above model has been subject to hundreds of simulations by changing parameter values. We show graphically that the nonlinear cobweb model with private storage can lead to a variety of price dynamics, including converging, quasi-periodic⁶ or chaotic time series⁷.

In figure 1, for the parameter values given above, the system converges. Production remains constant at about 1.21, as well as prices in each of the two seasons (1.61 and 2.0

⁶ Quasi-periodic series are series that look periodic on graphs, although a careful inspection of values of the series over time shows that they do not repeat themselves and are thus not periodic.

⁷ Notice all figures here picture the last 100 years of a 300 years simulation:

during seasons 1 and 2 respectively the difference corresponding to the cost of storage and discounting, the latter being high: 20%).

In figure 2, the values of a , α and β are changed. This change makes the dynamics switch from converging to a quasi-periodic time series. Although the time series in figure 2 looks periodic, in fact, the price values are always different, albeit close. The regular price time series of the model is not realistic, for the reason indicated above regarding periodicity.

In figure 3, we show simulations results with exactly the same parameters as for figure 1, except the interest rate, now 5% instead of 20%. We obtain a time series of prices that is now irregular, and looks random, although it is the consequence of the underlying dynamics of a purely deterministic model. The series we obtain are chaotic⁸. The nonlinear cobweb model developed earlier can thus lead to irregular price time series.

Finally, figure 4 shows another example of a chaotic series obtained from the model using different values for most parameters. Like in figure 3, we get an irregular pattern for price series, but unlike in figure 3, the price series shows frequent peaks and is thus asymmetric, as actual price series are. It could be argued though that the simulated series tend to have too many peaks and to look far more periodic than actual series.

The irregular price fluctuations in this model are generated by the “repelling” character of the unique fixed point of the system, the point where the supply and demand curves

⁸ The chaotic nature of the series can be assessed through the sensitivity to initial conditions or by plotting the distribution of prices (Alligood, Sauer and Yorke, 1997).

intersect. In the chaotic dynamics language, this point is a “sink” if the ratio of the absolute values of the demand and supply curves is less than one, a "source" otherwise.

In the case of a repelling fixed point, the original cobweb model leads to price sequences diverging up to infinity. This is indeed a major flaw of which Ezekiel (1938) was perfectly aware⁹. Here, it is corrected by the risk aversion, which stands as the force calling back the system toward equilibrium, just as a spring would do in a mechanical system¹⁰. With this addition of a risk avoiding behavior from the producer' part, for some values of parameters, the cobweb model results in a chaotic motion (Boussard, 1996).

Beside, with a supply curve depending upon marginal cost, but also upon previously experienced volatilities, the repelling character can be present even with $|\alpha/a| < 1$, although, of course, a low $|\alpha/a|$ decreases the chances of such an outcome. This remark illustrates one of the difficulties with this kind of chaotic model : the frequent absence of a connected domain, in the parameter space, where to locate the region corresponding to convergence. The output series can converge toward equilibrium for a given set of parameters values x_0 , be chaotic in the immediate ε vicinity of this set (that is, for all values x of parameters such that $|x_0 - x| < \varepsilon$ and converge again for some $x > x_0 + \varepsilon$). Simple statements such as “convergence occurs when $|\alpha/a| < 1$ ” are not possible. This does not facilitate deriving policy implications from such models: what is true for some parameter

⁹ Most of this reference is devoted to explain why prices cannot growth to infinity.

¹⁰ This literary description provides an intuitive account of what happens. A more rigorous mathematical treatment can be found in Boussard (1996)

values might not be true for others. It is all the more confusing as parameters are not known with certainty, and, in addition, are continuously changing (e.g., the marginal cost of production, not to say the demand price elasticity).

Finally, it must be stressed that, as in the theory of competitive storage (Wright and Williams 1982; D&L 1992), storage is essential to reproduce the main features of actual commodity series in the endogenous simulation framework. The same nonlinear cobweb model as above, but without private storage, consistently converges rapidly toward the equilibrium price.

3.3 Empirical Test

We put the model above to a test by following the method used by D & L (1992). For five specifications of the model, Table 2, shows coefficient of variation, autocorrelation of first rank, skewness, and kurtosis, in order to compare our results with both actual commodity price series and the results of the D&L (1992) model. For actual series, only the mean, minimum and maximum of the 13 commodities is presented. All values for each series can be found in D & L (1992).

With coefficients of variation ranging in the [0.0;0.75] interval, simulated series tend to be slightly more volatile than actual series, except sugar and cocoa, for which volatility is similar. The first order auto-correlation coefficients are lower for simulated annual prices¹¹ than for actual series: They are in the [0;0.70] range for simulated series compared to [0.62;0.91] for actual series. Similarly, second order autocorrelation

¹¹ Since there are two seasons in the model, we get two observations per years. An annual mean is computed, weighing prices by consumed quantities. The autocorrelation coefficient is then computed between these annual means – as it was, actually, for the time series observed by D&L.

coefficients tend to be lower for simulated series. Simulated series have significant positive skewness, thus exhibiting, like actual series, asymmetric fluctuations. Simulated series have positive kurtosis between 0 and 2, while in actual series, “heavy tails” is a well known feature of sugar and several other commodities price probability distributions¹². The above results also held when simulations were run over a shorter time span (100 years) in order to be consistent with actual series.

Additional simulation results are presented in the upper panel of Table 3, using two values for α , two values for a , allowing or not allowing inter-annual storage. Because combining all these parameter values one with each other results in a large number of situations, each of which corresponding to one run of the model, Table 3 does not list indicators for each of them. Instead, it gives the mean of selected indicators over all the runs corresponding to the listed values of the selected parameters¹³.

Overall, results from the test of this endogenous price fluctuation model are similar to those of the theory of competitive storage. D & L (1992) could not reproduce the high autocorrelation of actual series and had a maximum autocorrelation coefficient of first order of 0.48. Cafiero and Wright (2006) tried to improve this result of D & L (1992) and found an autocorrelation coefficient of 0.69 after reducing the cost of storage. Here we get a similar level of autocorrelation of first order, sometime rather high, for some values of the parameters. It must be stressed that our results regarding the main characteristics of the series generated by the model do depend upon the existence of storage.

¹² remember kurtosis is 3 and skewness is zero for a Gaussian variable, a kurtosis less than 3 meaning a “peaked, fat tail” distribution, a positive skewness meaning most of the distribution’s weight on the left of the mean.

¹³ More explanations are in the note of Table 3 and intervals for parameter values are given in Table 1.

3.4 The Effect of inter-annual storage on prices and production

Careful analyses of the effects of storage on prices, production and welfare have been conducted in the context of the theory of competitive storage (e.g., Wright and Williams 1982; 1984). We can use the above nonlinear cobweb model to start to assess the effects of storage on prices and production in this different theoretical framework. The lower panel of Table 3 includes simulation results when we do not allow inter-annual storage, which can be compared to the upper panel with results when we allow inter-annual storage. Table 3 shows that inter-annual storage has no clear effect on average prices – sometimes increasing, sometimes decreasing them, but clearly reduces the coefficient of variation. Inter-annual storage is therefore rather stabilizing in this context, at least whenever the system leads to a chaotic behavior: it should indeed be noted that the means in table 3 are computed over chaotic series only. In addition, the number of periodic or converging series is larger with than without inter-annual storage, which confirms the stabilizing character of the latter. This finding on the stabilizing effect of inter-annual storage is consistent with Lowry, Glauber, Miranda and Helmberger (1987) who conducted a similar analysis of the effects of inter-annual storage but in a seasonal model of the theory of competitive storage. In contrast with our results though, they also reached the conclusion that inter-annual storage leads to a gain in efficiency, in the form of a benefit to the consumer through lower prices, which we do not find here. Lowry, Glauber, Miranda and Helmberger (1987) also investigated how the cost of storage may influence prices. This is what we do in Table 4, where interest rates vary from 5 to 20%.

We find that increasing the rate of interest does not significantly change results for this range of parameters.

In Table 5, we vary the storage capacity of the storage firm. We find a slight increasing effect on prices and production. The only significant effect is on autocorrelation, very strongly reduced by a low storage capacity. This is not really surprising, since, for long storage has been held responsible for autocorrelation in commodity price series.¹⁴

This first assessment of the effects of storage on prices and production in the context of a nonlinear cobweb model with private storage leads to different results compared to those of several studies of the theory of competitive storage based on random shocks (Wright and Williams 1982; Lowry et al 1987, D&L 1992). These studies found that storage has overall a stabilizing effect on prices although it can occasionally lead to price peaks when stocks are empty. Overall, the effect of storage on prices and production is complex in a nonlinear cobweb model. As shown by results on inter-annual storage and on storage capacity, storage here appears to lead to lower price volatility, but not to lower prices. It does not seem to increase the efficiency of the system, which would allow lower average prices to the benefit of consumer. This conclusion, nevertheless, must be taken with caution, because the model does not allow reaping other potential benefits of price stabilization¹⁵: the latter may include easier access to credit, investments, and technical

¹⁴ Indeed, although not explicitly stated, a major motivation of D&L was to give evidence that storage was capable of transforming an iid process into an auto-correlated series.

¹⁵ They are listed by many authors, among whom Poulton *et al.* (2006), Timmer (1995), Chavas & Holt(1990), although no extensive review of the issue seems to exist.

progress, all features ignored in this model. This might explain why we did not find any consistent effect of storage on production.

4. Concluding remarks

We developed a dynamic agricultural commodity model with storage where no production shocks are needed to generate price fluctuations. The nonlinear cobweb model with storage empirically performs as well as the theory of competitive storage. It can reproduce the variability and asymmetry of actual commodity price series. It can also reproduce a positive autocorrelation of first rank, although it is not as high as that of actual series. Assumptions in this nonlinear cobweb model (i.e. risk aversion, adaptive expectations) stand in contrast with those used in the theory of competitive storage, and lead to somewhat different results on the impact of storage on prices. The presence of inter-annual storage and an increase in storage capacity lead to prices that are less volatile, but mean prices are not found to change significantly.

Our study findings need to be interpreted in the light of its limitations, some of which could be addressed in further research. It could be argued that the results of the nonlinear cobweb model are tied to one particular version of the cobweb model. While more work is needed modeling private storage in a nonlinear cobweb model, it should be noted that we have studied elsewhere other versions of the nonlinear cobweb model with private storage, for example with naïve instead of adaptive expectations¹⁶, with one agent producer-storer (Mitra 2001) and with a linear instead of a nonlinear demand curve (Mitra and Boussard, 2010). These different versions of the model could also lead to

¹⁶ The use of naïve expectations is supported by Chavas (2000): in an econometric model of US beef producers, close to half of producers behave in a way that is consistent with naïve expectations.

irregular price movements and reproduce some of the empirical characteristics of actual price series.

In addition, further analysis is required on the impact of inter-annual storage on welfare in the context of the cobweb model. Finally, while expectation errors are plausible, random disturbances undoubtedly exist in agricultural markets, and interact with other factors. Further research is needed to model agricultural price fluctuations by developing models which include *both* exogenous sources of fluctuations in the form of shocks as in the theory of competitive storage, and endogenous sources in the form of erroneous expectations and other behavioral or market characteristics (e.g. risk aversion) as in the cobweb model above.

Despite these limitations, our study contributes to the literature on agricultural commodity price fluctuations. Our results demonstrate the importance of assumptions regarding the information available to decision makers in deriving practical conclusions regarding the role of storage in price fluctuations. The nonlinear cobweb model with private storage developed in this paper performs as well as the theory of competitive storage in reproducing the empirical characteristics of agricultural commodity prices. Alongside models based on the theory of competitive storage, and despite the fact that all results are highly sensitive to parameters values, or assumptions regarding expectations, nonlinear cobweb models could therefore be used for the assessment of the impact of policy decisions such as trade reforms or price stabilization programs. Because of the sensitivity of the results to parameter values, more research is needed in this direction.

Indeed, from a purely mathematical point of view, chaotic series, although unpredictable with precision, by definition, stay on some attractor – a finite, not necessarily connected domain in \mathbb{R}^n . Therefore, for any practical purpose, the important thing is not the series itself, but the properties of the attractor. This is an area where the mathematical theory of attractors, although in its infancy, could have practical implications.

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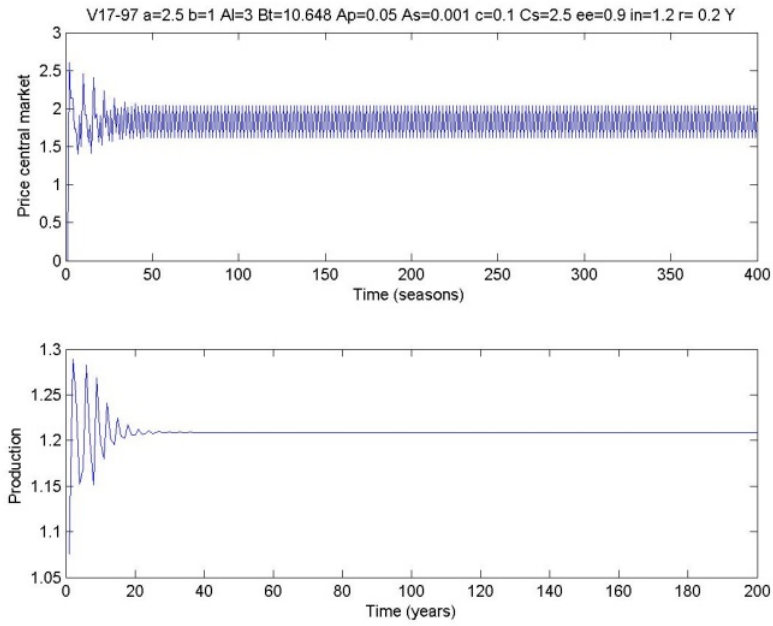


Figure 1: Converging seasonal equilibrium

Note: Parameter values are as follows:

$$\alpha_1 = \alpha_2 = 2; \beta_1 = \beta_2 = 0.001; a = 0.5; b = 3.16; A_f = 2; A_s = 1; \gamma_f = \gamma_s = 0.5; c = 0.01; S_{\max}; r = 0.05; T = 0.01$$

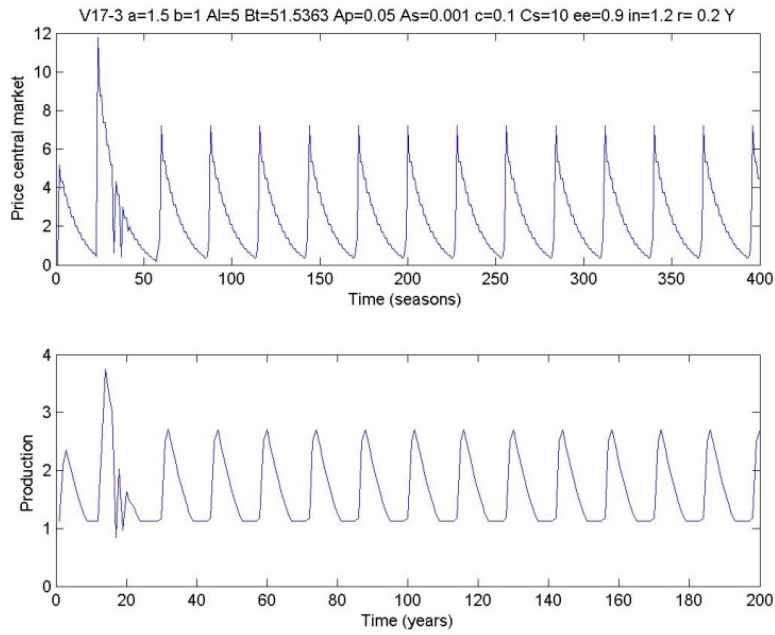


Figure 2: Quasi-periodic motion

Note: Parameter values are as follows:

$$\alpha_1 = \alpha_2 = 2; \beta_1 = \beta_2 = 0.001; a = 1; b = 1; A_f = 2; A_s = 1; \gamma_f = \gamma_s = 0.5; c = 0.01; S_{\max}; r = 0.2; T = 0.01$$

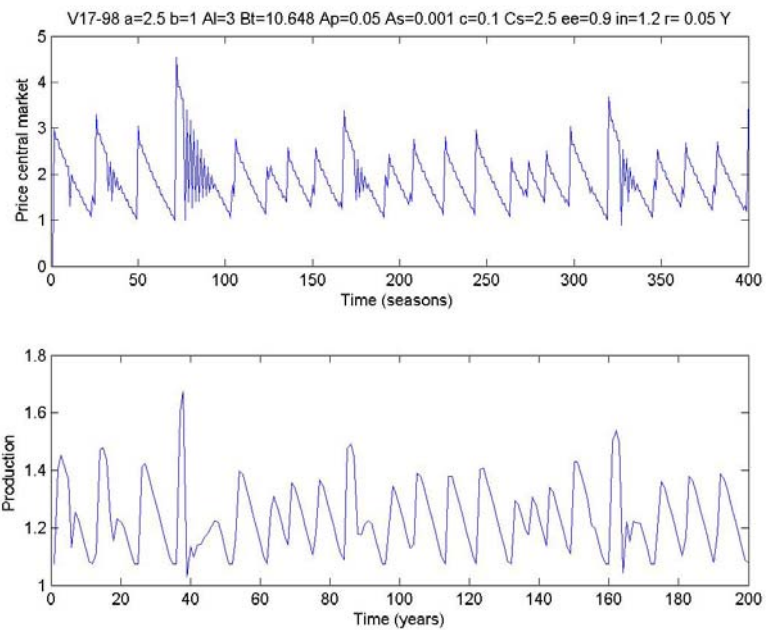


Figure 3: Chaotic motion

Note: Parameter values are as follows:

$$\alpha_1 = \alpha_2 = 3; \beta_1 = \beta_2 = 10.648; a = 2.5; b = 1; A_f = 0.05; A_s = 0.001; \gamma_f = \gamma_s = 0.5; c = 0.01; S_{\max} = 2.5; r = 0.05$$

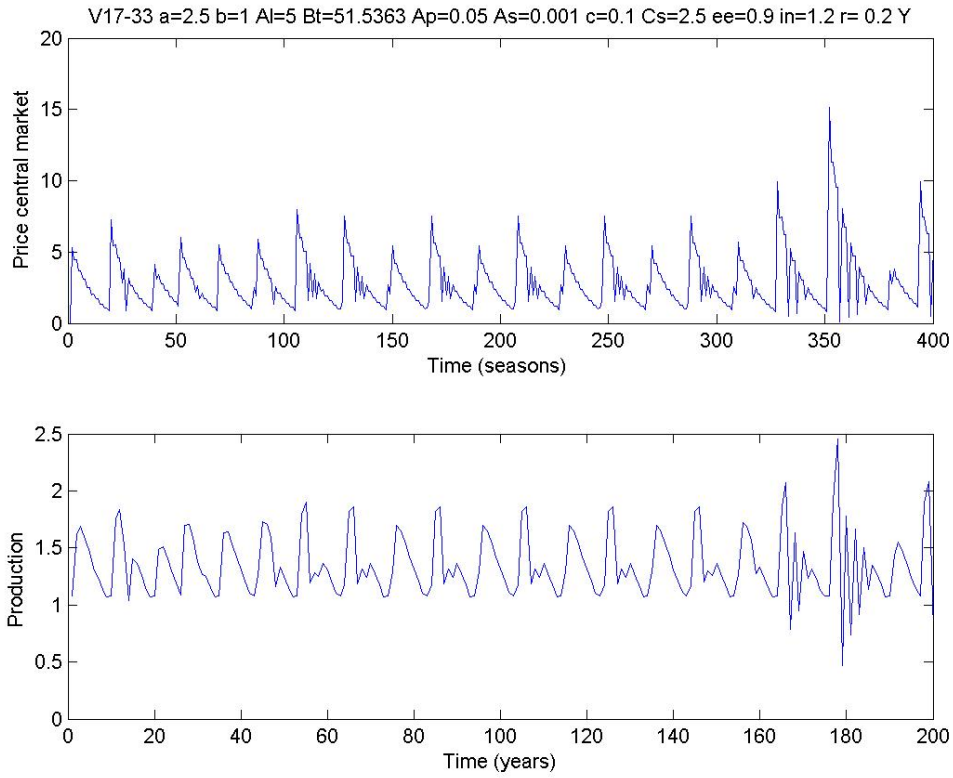


Figure 4: Another example of chaotic motion

Note: Parameter values are as follows:

$$\alpha_1 = \alpha_2 = 5; \beta_1 = \beta_2 = 51.54; a = 2.5; b = 1; A_f = 0.05; A_s = 0.001; \gamma_f = \gamma_s = 0.5; c = 0.01; S_{\max} = 2.5; r = 0.2; T$$

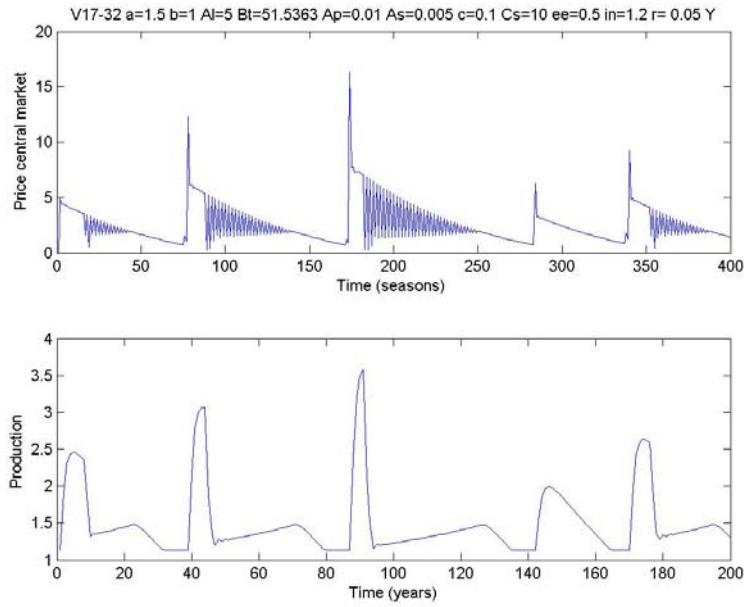


Figure 5: Chaotic motion with low storage capacity

Note: Parameter values are as follows:

$$\alpha_1 = \alpha_2 = 1.5; \beta_1 = \beta_2 = 51.5; a = 1.5; b = 1; A_f = 0.01; A_s = 0.005; \gamma_f = \gamma_s = 0.5; c = 0.01; S_{\max} = 10; r = 0.05; Y$$

Table 1: Parameter Specifications

Variable	Definition	Value or interval of values
α_i	Log demand slope in season i	[1 -3]
β_i	Log demand intercept in season i	[0.00001 -0.1]
a	Log supply slope	[0.5 - 1]
b	Log supply intercept	[2.08 - 3.16]
A_f	Coefficient of absolute risk aversion (farmer)	2
A_s	Coefficient of absolute risk aversion (storage firm)	[0.5 - 2]
γ_f	Coefficient of expectation correction (farmer)	[0.2 - 0.8]
γ_s	Coefficient of expectation correction (storage firm)	[0.2 - 0.8]
c	Physical storage cost	[0.01 - 0.01]
S_{max}	Storage capacity	[0.0025 - 1]
r	Interest rate	[5% - 20%]
T	Transportation and handling costs	1

For simplicity, we assume that the parameters of the demand curve are the same in both seasons.

Table 2: Comparison of annual price series from nonlinear cobweb model with storage to actual series and D&L (1992) series

	Mean	Coefficient of variation	AC 1	Skewness	Kurtosis
Simulated series of nonlinear cobweb with storage (1)					
(1) Figure 1	1.8	1.00E-10	-0.01	16.2	-0.76
(2) Figure 2	4.82	0.7	0.29	1.6	0.12
(3) Figure 3	3.65	0.78	0.6	1.96	0.28
(4) Figure 4	2.03	0.40	0.53	1.82	-0.80
(5) Figure 5	3.28	0.56	1.18	3.26	0.22
Actual series					
Mean		0.39	0.8	1.06	2.38
Minimum		0.17	0.62	0.04	-0.98
Maximum		0.6	0.91	3.24	16.52
Simulated series of Deaton & Laroque's model					
(a)		0.1	0.08	0.43	-0.29
(b)		0.5	0.48	1.99	5.5
(c)		0.1	0.08	0.61	0.17
(d)		0.53	0.33	3.41	24.22
(e)		0.53	0.29	3.15	16.43

AC 1 stands for autocorrelation of first rank

Actual series are for 12 commodities as reported in D&L (1992)

Table 3: Simulations of the nonlinear cobweb model with storage

	$\alpha=5$ $a=2.5$	$\alpha=5$ $a=1.5$	$\alpha=3$ $a=2.5$	$\alpha=3$ $a=1.5$
<u>Cobweb model with intra-annual storage only</u>				
Mean central market Price season 1	3.18	3.16	2.06	2.40
Mean central market Price season 2	2.34	1.89	1.85	2.05
Production (x 1,000)	1.35	1.49	1.27	0.22
Stock in season 1 (x 1,000)	3.02	3.39	2.47	1.80
Stock in season 2 (x 1,000)	3.63	4.03	3.08	2.40
Annual price coefficient of variation	0.58	0.61	0.39	0.43
Production coefficient of variation	0.19	0.29	0.13	0.22
Mean AC 1 of annual price	0.49	0.36	0.63	0.34
<i>Number of simulations</i>	32	32	32	32
<i>Percentage of cases with chaotic dynamics or long period</i>	56%	94%	13%	50%
<u>Cobweb model with intra and inter-annual storage</u>				
Price in Season 1	4.00	3.56	2.01	1.96
Price in season 2	3.70	3.49	1.78	1.73
Production (x 1,000)	1.50	1.95	1.25	1.41
Stock in season 1 (x 1,000)	0.00	0.00	1.79	0.00
Stock in season 2 (x 1,000)	0.80	1.14	2.39	0.69
Annual price coefficient of variation	1.10	1.12	0.31	0.54
Production coefficient of variation	0.30	0.49	0.11	0.23
AC 1 of Annual price	-0.40	-0.43	0.60	-0.50
<i>Number of simulations</i>	32	32	32	32
<i>Percentage of cases with chaotic dynamics or long period</i>	50.0%	31.3%	0.0%	40.6%

For each cell, the mean of the relevant indicator is given for the indicated number of simulations, extracted from a set of 192, with three values for the slope of demand curve, and two values for the slope of supply curve, the interest rate, the elasticity of expectation, the producer risk aversion coefficient, and the storage capacity.

Only chaotic results have been included in the results above (Converging or periodic series are ignored).

Table 4 : Effects of changes in the interest rate on prices and production (1)

Rate of interest (%)	5	20
Price in Season 1 (summer)	2.81	2.89
Price in Season 2 (winter)	1.94	1.93
Producer Price	1.93	1.92
Production	1.39	1.43
Stock in Season 1 (summer)	3.68	1.68
Stock in Season 2 (winter)	4.30	2.32
Coefficient of Variation of Annual Central Price	0.50	0.62
AC 1 of Annual Central Price	0.47	0.31
<i>Number of simulations</i>	54	54
<i>Percentage of cases with chaotic dynamics or long period</i>	74%	52%

We vary the value of the interest rate r . For each value of r , we run about 30 simulations changing the values of storage capacity, expectation elasticity, the demand and supply slopes. Only chaotic results have been included in the results above (Converging or periodic series are ignored).

Table 5 : Effects of changes in the storage capacity on prices and production (1)

Storage capacity	2.5	10
Price in Season 1 (summer)	1.87	2.00
Price in Season 2 (winter)	2.98	2.71
Producer Price	1.86	1.99
Production	1.38	1.43
Stock in Season 1 (summer)	1.25	4.37
Stock in Season 2 (winter)	1.85	5.03
Coefficient of Variation of Annual Central Price	0.51	0.58
AC 1 of Annual Central Price	0.17	0.62
<i>Number of cases</i>	64	64
<i>Percentage of cases with chaotic dynamics or long period</i>	55%	52%

We vary the value of the storage capacity. For each value of the storage capacity, we run about 30 simulations changing the values of the interest rate, expectation elasticity, the demand and supply slopes. The table includes mean values for all these simulations.

Only chaotic results have been included in the results above (Converging or periodic series are ignored).